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On the string description of confinement

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Abstract: A non supersymmetric string background, directly derived from the string soft dilaton theorem, is used to compute, in the semiclassical approximation, the expectation value of Wilson loops in static gauge. The resulting potential shares common features with the one obtained through Schwarzschild-anti de Sitter space-time metrics In particular a linear confining potential appears naturally.

Keywords: Confinement, Bosonic Strings.

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Contents

1. Introduction

The simplest candidate for a string representing pure Yang Mills theory, is a non critical string with *curved Liouville* [8] action:

$$L = \left(\frac{a(\phi)}{l_s}\right)^2 (\partial x)^2 + (\partial \phi)^2 + T(\phi) + \Phi(\phi) R_2 \,, \tag{1.1}$$

where x stands for the four dimensional space-time coordinates, ϕ for the Liouville field and $T(\phi)$ and $\Phi(\phi)$ for the closed tachyon and dilaton backgrounds. The factor $a(\phi)/l_s$ would be interpreted as an effective *running* string tension for the four dimensional non critical string [9]. The scale l_s plays then the rôle of a *bare* string tension. The space-time metric associated to the preceding action (1.1) is:

$$ds^2 = a(\phi)^2 dx^2 + l_c^2 d\phi^2 \,, \tag{1.2}$$

where we have introduced an extra scale l_c for dimensional reasons. The physical meaning of this scale will become clear as we proceed.

The physical backgrounds $a(\phi)$, $T(\phi)$ and $\Phi(\phi)$ should be restricted by demanding the vanishing of the two dimensional sigma model beta functions.

The soft dilaton theorem [1] for vanishing dilaton tadpoles (owing to conformal invariance) reads:

$$\left(\sqrt{\alpha'} \frac{\partial}{\partial \sqrt{\alpha'}} - \frac{1}{2}(d-2) g \frac{\partial}{\partial g}\right) A(p_1, p_2, \ldots, p_n) = 0 \,. \tag{1.3}$$

This equation (1.3) can be considered, for $d = 4$, as a renormalization group equation[1] with:

$$\sqrt{\alpha'} \frac{\partial g}{\partial \sqrt{\alpha'}} = \beta(g) = -g \,. \tag{1.4}$$

[1]In closed string field theory the soft dilaton theorem becomes equivalent to the invariance of the string field action under space-time dilatations and changes of the string coupling (see [6] and [13])

Using the definition of g in terms of the dilaton field:

$$e^{\Phi} = g \tag{1.5}$$

and interpreting, as discussed above, $\sqrt{\alpha'}$ as $l_s/a(\phi)$ we get from $(1.4)^2$:

$$\Phi = \log\left(\frac{a(\phi)}{l_s}\right) \tag{1.6}$$

We will take (1.6) as the starting point to determine the backgrounds in (1.1). Vanishing of the sigma model beta functions (to first order) leads to the following solution:

$$ds^2 = \phi\, d\vec{x}^2 + l_c^2 d\phi^2 \tag{1.7}$$

$$\Phi(\phi) = \frac{1}{2}\log(\phi) \tag{1.8}$$

where we have fine tunned the closed string tachyon vacuum expectation value to compensate the central charge deficiency in the dilaton beta function equation. A first analysis of the stability of this solution was presented in [2].

In this letter we will consider the problem of confinement for the background metric (1.7), by explicit computation in the semiclassical approximation of the Wilson loop vacuum expectation value.

2. Wilson Loop

An interesting feature of the preceding metric (1.7) is the existence of a *naked* singularity at $\phi = 0$. From (1.5) and (1.7) this corresponds to the weakly coupled regime of the dual gauge theory. The boundary of space-time (1.7) is at $\phi = \infty$.

In order to compute the Wilson loop we will follow a Nambu-Goto semiclassical approximation in static gauge [7, 11]. We then indentify

$$\begin{aligned} t &= \tau \\ x &= \sigma \end{aligned} \tag{2.1}$$

The induced metric on the world sheet will then be, for static configurations,

$$ds^2 = \phi\, d\tau^2 + (\phi + l_s^2\phi'^2)d\sigma^2 \tag{2.2}$$

and the action reads:

$$S = \frac{T}{{l_s}^2}\int_0^1 d\sigma\sqrt{\phi(\phi + l_c^2\phi'^2)}\,. \tag{2.3}$$

[2]After these identifications (1.3) becomes equivalent [3] to the holographic renormalization group [4]

We will consider U-shape string configurations with Dirichlet boundary conditions at the codimension one hyersurface $\phi = \Lambda$. Denoting by ϕ_0 the tip of the U-shape string we get:

$$\frac{\phi^2}{\sqrt{\phi(\phi + l_c^2 \phi'^2)}} = \phi_0 \,. \tag{2.4}$$

From (2.4) we easily get for a loop of size L the relation:

$$L = 2 l_c \phi_0^{1/2} \int_1^{\sqrt{\Lambda/\phi_0}} \frac{d\xi}{\sqrt{\xi^4 - 1}} \,. \tag{2.5}$$

The action is then given by:

$$S = \frac{2 T l_c \phi_0^{3/2}}{l_s^2} \int_1^{\sqrt{\Lambda/\phi_0}} \frac{\xi^4 d\xi}{\sqrt{\xi^4 - 1}} \,. \tag{2.6}$$

Please notice that the integral (2.6) is divergent in the limit $\Lambda = \infty$. In terms of elliptic functions we get:

$$L = \frac{2 l_c \phi_0^{1/2}}{\sqrt{2}} F\left(\cos^{-1}\sqrt{\frac{\phi_0}{\Lambda}}, \frac{1}{\sqrt{2}}\right) \tag{2.7}$$

$$S = \frac{2 T l_c \phi_0^{3/2}}{l_s^2} \left(\frac{1}{3\sqrt{2}} F\left(\text{arcos}\sqrt{\frac{\phi_0}{\Lambda}}, \frac{1}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{\frac{\Lambda}{\phi_0}}\sqrt{\left(\frac{\Lambda^2}{\phi_0^2} - 1\right)}\right) . \tag{2.8}$$

From the first equation (2.7) we can read the relationship between $L/\sqrt{\Lambda}$ and $\sqrt{\phi_0/\Lambda}$ as plotted in figure 1.

There are several interesting features. First of all the existence of a maximun indicates that the size of the loop L should be necessarily smaller that $\sqrt{\Lambda}$ in l_c units. Secondly for a given L we get two different U-shape string configurations (see figure 2).

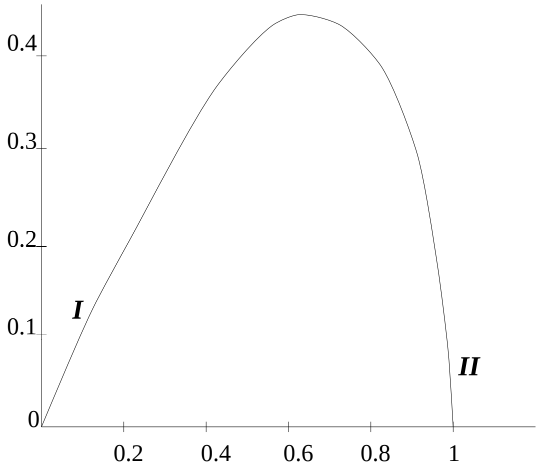

Figure 1: $L/(\sqrt{\Lambda}) l_c$ versus $\sqrt{\phi_0/\Lambda}$.

This phenomenon is similar to the one found in [10] for Schwarzschild-anti de Sitter (S-AdS) space-time (see figure 3 for a qualitative comparison). In fact in [10] a maximum was also obtained (in the context of the S-AdS space-time) as well as two possible U-string configurations for each loop size L. However it is important to stress that in our case this phenomenon

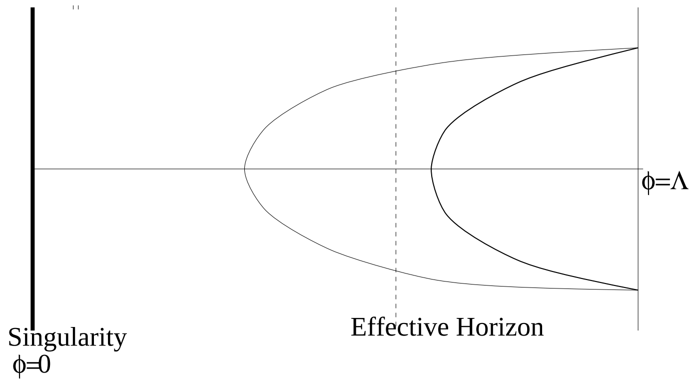

Figure 2: The two U-shaped string configurations corresponding to the same value of L.

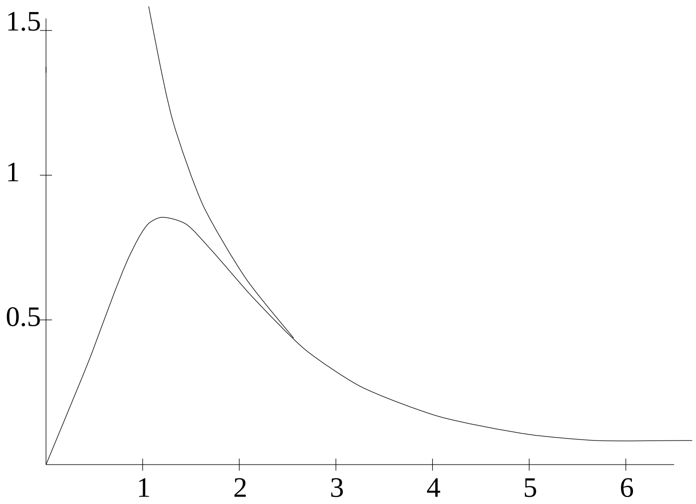

Figure 3: Relation between L and the location of the tip of the U-String configuration for SadS space-time [10]. Also included the curve for AdS space-time.

depends on having the cutoff Λ. In fact the naïve limit $\Lambda = \infty$ would produce the relationship:

$$L = \frac{2l_c\phi_0^{1/2}}{\sqrt{2}} F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) \tag{2.9}$$

Relying upon the similarity with the Schwarzschild-anti de Sitter example we can think of the vertical dashed line in figure 3 as a sort of *effective horizon* covering the singularity at $\phi = 0$. Please notice that the limit $\sqrt{\phi_0/\Lambda} = 0$ corresponds to pushing this horizon on top of the singularity itself. Hence it looks as if the system sort of regularizes the singularity through a non trivial dependence of ϕ_0 on Λ.

Coming back to figure 3 let us briefly recall the physical meaning of the two branches. Region I corresponds (see [10]) to a confining behavior for the quark potential while region II describe the Coulomb phase typical of AdS_5. Qualitatively in the deep region II, Wilson loops are defined in terms of electric flux tubes that enter only in the asymptotically AdS region reproducing the result of $N = 4$ super Yang- Mills.

In our case we can also differenciate between regions I and II of figure 2. The static potential in region I corresponding to $\phi_0 \ll \Lambda$ is given by:

$$V = \frac{L^3}{24K^2 l_s^2 l_c^2} + \frac{4}{3}\frac{l_c}{l_s^2}\Lambda^{3/2}\,, \tag{2.10}$$

(where $K \equiv K(k = 1/\sqrt{2})$ is the complete elliptic integral of the first kind), This physically means an *overconfining* L^3 potential between static probes. It is important to stress that the divergent part in (2.10) cannot be directly interpreted as a *mass renormalization.*

In the region II for ϕ_0 close to Λ and $L \ll \Lambda$, in l_c units,[3] we get:

$$V = \frac{L\Lambda}{l_s^2}\left(1 + \frac{\pi\sqrt{2}}{2K}\right) \tag{2.11}$$

i.e. a linear confining behavior. Using equations (1.5) and (1.8) the leading term of the potential can be rewritten in terms of the *runnig* effective coupling g as:

$$V = \frac{Lg^2}{l_s^2}\left(1 + \frac{\pi\sqrt{2}}{2K}\right) \tag{2.12}$$

i.e. a string tension of the order g^2/l_s^2.

Given a physical value of L, it is possible, for each Λ, to determine which of the two allowed values of ϕ_0 (corresponding to region I or region II, respectively), gives smaller potential energy. In the region in which the approximations are valid it can be written

$$L = \epsilon l_c \Lambda^{1/2} \tag{2.13}$$

in such a way that

$$\frac{V_{II}}{V_I} = \frac{\epsilon}{\epsilon^3/24K^2 + 4/3}\,. \tag{2.14}$$

3. Comments

It is natural to expect a renormalization group equation for the Wilson loop of the type:

$$\left(\lambda\frac{\partial}{\partial\lambda} + \beta(g)\frac{\partial}{\partial g}\right) W(C) = 0 \tag{3.1}$$

for dilatations $x \to \lambda x$. The Wilson loop we get satisfy this equation for the beta function (1.4). This is the beta function governing the string field theory. It would be extremely interesting to unravel the relation between equations of type (3.1) and the loop equations.

[3] Notice that here l_c is playing the role of Λ_{QCD}.

In summary in this letter we have presented a gravitational framework to study confinement in non supersymmetric Yang Mills. The gravitational background is dictated by the closed-open relation in string theory as encoded in the soft dilaton theorem. The dynamics of Wilson loop on this background shares many of the features previously found in descriptions of confinement using generalizations of AdS/CFT correspondence [12, 10, 5].

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JHEP05(2000)012

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